Chapter 5

Atmospheric Boundary Layer, Ground Surface Process and Ground Temperature

The integration time of a cloud model is short ranging from few to ten hours. Therefore, in the model, most of phenomena provided by the atmospheric processes. When energy flux from the ground surface (land or sea) is large in a cloud / precipitation system as in the case of a thunderstorm in summer or a snow cloud over Japan Sea in winter, however, ground surface process is substantially important for the development of the system.
Following processes are considered as the ground surface processes and they are expressed by simple bulk methods.

- vertical one-dimensional diffusion in the atmospheric boundary layer
- fluxes of sensible heat and latent heat from the ground to atmosphere
- momentum flux (friction of the ground surface)
- soil temperature variation

In CReSS, these processes are expressed with vertical one-dimensional equations. We adopt the ground surface flux with a bulk method and soil temperature forecasting which are used in JSM (Japan Spectral Model; Segami et al., 1989). Complicated soil and vegetation models are incorporated in some cloud models (e.g. RAMS, ARPS). However, they are important only when integration for a long time (month order) in a wide area (e.g. the region covering all the Eurasian Continent) is conducted. Therefore, the complicated soil and vegitation models will be added to ***CReSS*** in the future.

5.1 Basic Theories of Atmospheric Boundary Layer

Here, we summarize briefly basic theories for the atmospheric boundary layer processes[1].

5.1.1 Structure of Atmospheric Boundary Layer

Atmosphere is bordered on the lower boundary by the ground surface. Generally, effects of the ground surface which includes various conditions like land and sea become stronger as the altitude becomes lower. The effects exist from the surface to about 1-2 km high though the depth is affected with conditions of the ground surface and atmosphere. Such atmospheric layer which is thermodynamically and dynamically affected with the ground surface is called atmospheric boundary layer or planetary boundary layer; PBL.

Atmospheric boundary layer is devided into two layers; surface boundary layer which ranges from the surface to 20-50 m high and is affected by the ground surface, and Ekman layer or outer boundary layer above the surface boundary layer. Atmosphere above the atmospheric boundary layer is called free atmosphere.

The surface boundary layer has a property that vertical fluxes of sensible heat, latent heat, momentum and so on are uniform vertically and equal to their values on the ground. In other words, the atmospheric layer where these vertical fluxes are equal to the values on the ground can be called the surface boundary layer. The surface boundary layer is sometimes called constant flux layer. Because momentum flux is vertically constant, stress of turbulence is vertically constant in the surface boundary layer, that is, wind direction is also constant vertically.

5.1.2 Parameterization of Turbulence Transportation

Motions of various scale exist in the atmosphere. Among them, motion which can be expressed on the grid points of a numerical model is called grid-scale motion or mean motion. Motion which has a scale smaller than a grid size is called subgrid-scale motion or eddy motion.

To classify these motions, physical quantity A (e.g. wind speed, temperature, mixing ratio of water vapor) is devided into two quantity; averaged quantity which can be expressed on the grid points and deviation from that, as follows.

$$A = \bar{A} + A'' \tag{5.1}$$

There are some ways to average physical quantities, we will not mentione about them here. Average of deviation and that of product of two physical quantities are expressed as follows;

$$\overline{A''} = 0 \tag{5.2}$$
$$\overline{AB} = \bar{A}\bar{B} + \overline{A''B''} \tag{5.3}$$

The average of the pruducts of some quantities is not always equal to the product of averages of each quantity, and the second term of the equation above appears. This equation is applied to the x component of equations of motion. Considering an uncompressed fluid ($\rho = const.$), each term of the equation can be expressed with the average and deviation from that as follows;

$$\begin{aligned}\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} - f\bar{v} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} \\ -\frac{1}{\rho}\left(\frac{\partial}{\partial x}\rho\overline{u''u''} + \frac{\partial}{\partial y}\rho\overline{u''v''} + \frac{\partial}{\partial z}\rho\overline{u''w''}\right) + \nu\nabla^2\bar{u}.\end{aligned} \tag{5.4}$$

[1]Introduction to dynamic meteorology (Ogura, 1978) and Atmospheric Science Course 1 - Atmosphere near the ground surface (Takeuchi and Kondo, ????) are mainly reffered here.

The terms of $-\rho\overline{u''u''}, -\rho\overline{u''v''} and - \rho\overline{u''w''}$ express the stress of turbulence and are called eddy stress or Reynolds stress. These stresses can be regarded as a momentum transportation, and the momentum transportation by eddy is the stress.

Similarly, equations of potential temperature and mixing ratio of water vapor are as follows.

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{u}\frac{\partial\bar{\theta}}{\partial x} + \bar{v}\frac{\partial\bar{\theta}}{\partial y} + \bar{w}\frac{\partial\bar{\theta}}{\partial z} = -\frac{\partial}{\partial x}\overline{u''\theta''} - \frac{\partial}{\partial y}\overline{v''\theta''} - \frac{\partial}{\partial z}\overline{w''\theta''} \tag{5.5}$$

The quantities with a prime cannot be expressed with the grid points. It is neccessary to express them with quantities on the grid points (averaged quantities) for including effects of momentum transportation by eddy which is smaller than the grid size. A problem of turbulent parameterization is to decide the way to express deviation of quantity by eddy with prime on the grid points. Note that definitions of quantities with $^{-}$ used in this chapter and in the other chapter (e.g. Chapter 2) are different.

5.1.3 Mixing Length Theory

As mentioned in the chapter of turbulence, atmospheric motions should have a motion which is smaller than the grid size. Therefore, the equation system is not closed due to correlation quantities of deviation as in (5.4) and (5.5). This is called closure problem and the terms of transportation by turbulence must be expressed as functions of quantities on grid points to close the equation system. One of the turbulent parameterization methods is mixing length theory by Prandtl (1925). This is explained by an analogy of kinetic theory of gases as follows.

We assume that the mean vertical speed of air mass at the altitude of $z - l''$ is $\bar{u}(z - l'')$, and gives momentum to the environment when the air mass moves upward for l'' reaching to the altitude of z. The variation of the speed is

$$u'' = \bar{u}(z - l'') - \bar{u}(z) \cong -l''\frac{\partial\bar{u}}{\partial z}. \tag{5.6}$$

Raynolds tension is

$$\tau_{zx} = -\rho\overline{u''w''} = \rho\overline{l''w''}\frac{\partial\bar{u}}{\partial z} = \rho K_{mz}\frac{\partial\bar{u}}{\partial z} \tag{5.7}$$

where

$$K_{mz} = \overline{l''w''} \tag{5.8}$$

is eddy viscosity coefficient.

Considering fluid with neutral stratification, the effect of buoyancy can be neglected, and it can be assumed $|w''| \sim |u''|$. We can assume the following relation.

$$w'' \sim u'' \cong -l''\frac{\partial\bar{u}}{\partial z} \tag{5.9}$$

Then,

$$\tau_{zx} = -\rho\overline{u''w''} = \rho\overline{l''^2}\frac{\partial\bar{u}}{\partial z}\left|\frac{\partial\bar{u}}{\partial z}\right|. \tag{5.10}$$

and,

$$K_{mz} = \overline{l''^2} \left| \frac{\partial \bar{u}}{\partial z} \right| . \tag{5.11}$$

Using these equations,

$$-\frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{u''w''} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_{mz}\frac{\partial \bar{u}}{\partial z}\right) \tag{5.12}$$

$$-\frac{\partial}{\partial z}\overline{w''\theta''} = -\frac{\partial}{\partial z}\left(K_{hz}\frac{\partial \bar{\theta}}{\partial z}\right) \tag{5.13}$$

are gained.

As mentioned above, the transportation of physical quantities with turbulence could be explained by using mean field based on an analogy of kinetic theory of gases. That is called Prandtl's mixing length theory. It is difficult to apply generally this theory for turbulent transportation. For the surface boundary layer, we can apply the theory as a basic theory to calculate ground surface flux. The other theories are needed to solve a closure problem for the whole of the atmospheric boundary layer including Ekman layer.

5.1.4 Surface Boundary Layer

The surface boundary layer is called constant flux layer where a value of vertical momentum flux at all altitudes is equal to that of the ground surface, and stress is constant. Therefore, wind direction is constant in the surface boundary layer. We set that the direction of an averaged wind is the x direction. Considering (5.10),

$$\frac{\partial \bar{u}}{\partial z} = \frac{1}{l}\sqrt{\frac{\tau_{zx}}{\rho}} \tag{5.14}$$

where $l = \sqrt{\overline{l''^2}}$. The size of turbulence which transports momentum within the surface boundary layer is restricted by the ground surface. The mixing length l is assumed as follows;

$$l = k\ z \tag{5.15}$$

where z is the distance from the surface and k is nondimensional quantity called Karman constant. k is decided as $k = 0.4$ from wind tunnel experiments and observations[2].

We define u_* as follows;

$$u_*^2 \equiv \frac{\tau_{zx}}{\rho} = -\overline{u''w''}. \tag{5.16}$$

This is called friction velocity and has a dimension of velocity, corresponding to the scale of turbulent strength.

Observations showed that the amount of vertical transportation of momentum (or stress) is constant to altitudes within the surface boundary layer. Using these equations,

[2]Karman constant is decided as $k = 0.4 \pm 0.01$ by simultaneous observations of Raynolds stress $\tau = -\rho\overline{u''w''}$ and vertical profile of wind speed (Meteorology of water environment, p99, Kondo, ????).

$$u_* = \sqrt{\frac{\tau_{zx}}{\rho}} = kz\frac{\partial \bar{u}}{\partial z} = const. \tag{5.17}$$

is gained. Integrating this equation for z,

$$\bar{u} = \frac{u_*}{k}\ln z + C. \tag{5.18}$$

Since the actual ground surface has various irregurality, it can not say that $\bar{u} = 0$ at $z = 0$. Then, the constant of integration C is decided as the value which makes $\bar{u} = 0$ at $z = Z_{0m}$.

$$\bar{u} = \frac{u_*}{k}\ln\frac{z}{Z_{0m}} \tag{5.19}$$

This is the logarithmic profile of wind speed. Z_{0m} means the extent of roughness on the ground and is called aerodynamical roughness parameter (or length) or ground surface roughness for wind profile. The logarithmic profile expressed with this equation corresponds well with the actual wind profile when the atmosphere has a neutral stratification.

Eddy viscosity coefficient in the surface boundary layer K_{mz} can be expressed by (5.11) and (5.17) as follows;

$$K_{mz} = (kz)^2\frac{u_*}{kz} = kzu_*. \tag{5.20}$$

5.1.5 Vertical Flux

Momentum flux and Bulk coefficient

In the surface boundary layer, vertical flux is constant for altitude and it is enough to estimate flux at an altitude in the surface boundary layer for the estimation of flux on the ground. Momentum flux, i.e., wind stress on the ground is as follows;

$$\tau_0 = -\rho\overline{u''w''} = \rho u_*^2 = \rho\bar{u}^2\left[\frac{k}{\ln\dfrac{z}{Z_{0m}}}\right]^2 = \rho C_m\bar{u}^2 \tag{5.21}$$

$\bar{u}$ can be gained by measuring average wind speed at an altitude (usually 10 m) in the surface boudnary layer. C_m defined as

$$C_m = \left[\frac{k}{\ln\dfrac{z}{Z_{0m}}}\right]^2 \tag{5.22}$$

is nondimensional quantity and called drag coefficient or bulk transfer coefficient.

Potential Temperature Flux

In the surface boundary layer, it can be considered that the amount of transportations (fluxes) of sensible heat and latent heat H_s, H_L are constant at all altitudes. If turbulence is decided by u_* and z in the surface boundary layer, equation which appears in (5.13);

$$K_{mz}\frac{\partial\bar{\theta}}{\partial z} = \overline{w''\theta''} \tag{5.23}$$

is expressed as follows by using eddy viscosity coefficient (**??**) as in the case of momentum;

$$\frac{\partial\bar{\theta}}{\partial z} = \frac{\overline{w''\theta''}}{kzu_*} \tag{5.24}$$

Sensible heat flux from the ground H_s $[\mathrm{J\ m^{-2}\ s^{-1}}]$ is offered as follows;

$$H_s = -\rho C_p\overline{w''\theta''} \tag{5.25}$$

where potential temperature scale θ_* which is defined by

$$\theta_* = \frac{\overline{w''\theta''}}{u_*} \tag{5.26}$$

and potential temperature flux is expressed as follows;

$$\frac{H_s}{\rho C_p} = -\overline{w''\theta''} = -kz\frac{\partial\bar{\theta}}{\partial z}u_* = -\theta_* u_* \tag{5.27}$$

θ_* defined here is constant in the surface boundary layer. Therefore,

$$\frac{\partial\bar{\theta}}{\partial z} = \frac{\theta_*}{kz} \tag{5.28}$$

is obtained. By integrating this vertically and defining Z_{0h} as the altitude of $\bar{\theta} = \theta_G$ as in the case of momentum,

$$\bar{\theta} - \theta_G = \frac{\theta_*}{k}\ln\frac{z}{Z_{0h}} \tag{5.29}$$

is gained. Potential temperature flux is expressed with this as follows;

$$\frac{H_s}{\rho C_p} = -\overline{w''\theta''} = -\theta_* u_* = -\frac{k}{\ln\dfrac{z}{Z_{0h}}}\frac{k}{\ln\dfrac{z}{Z_{0m}}}(\bar{\theta} - \theta_G)\bar{u} \tag{5.30}$$

where

$$\frac{H_s}{\rho C_p} = -\frac{k}{\ln \frac{z}{Z_{0h}}} \frac{k}{\ln \frac{z}{Z_{0m}}} (\bar{\theta} - \theta_G)\bar{u} = -C_h(\bar{\theta} - \theta_G)\bar{u}. \tag{5.31}$$

Bulk coefficients of sensible heat and potential temperature can be defined as follows;

$$C_h = \frac{k}{\ln \frac{z}{Z_{0h}}} \frac{k}{\ln \frac{z}{Z_{0m}}}. \tag{5.32}$$

Mixing Ratio of Water Vapor Flux

Latent heat flux H_L can be regarded to be constant in the surface boundary layer,

$$H_L = {}_lE = -\mathcal{L}_v \overline{w'' q_v''} \tag{5.33}$$

and

$$\frac{\partial \bar{q}_v}{\partial z} = \frac{\overline{w'' q_v''}}{kzu_*} \tag{5.34}$$

where L is latent heat of evaporation. Flux of mixing ratio of water vapor is expressed as follows.

$$E = -\overline{w'' q_v''} = -kzu_* \frac{\partial \bar{q}_v}{\partial z} \tag{5.35}$$

In the same way, we define the scale of mixing ratio of water vapor (friction mixing ratio) q_{v*} as

$$q_{v*} = \frac{\overline{w'' q_v''}}{u_*} \tag{5.36}$$

and flux of mixing ratio of water vapor is expressed as

$$E = -\overline{w'' q_v''} = -kz \frac{\partial \bar{q}_v}{\partial z} u_* = -q_{v*} u_* \tag{5.37}$$

where q_{v*} defined here is also constant in the surface boundary layer. By integrating this vertically and defining an altitude Z_{0h} of which $\bar{q}_v = q_{vsfc}$ as same as the case of momentum and potential temperature,

$$\bar{q}_v - q_{vsfc} = \frac{q_{v*}}{k} \ln \frac{z}{Z_{0h}} \tag{5.38}$$

is given. This equation is expressed with Mixing ratio of water vapor as follows;

$$-\overline{w''q_v''} = -q_{v*}u_* = -\frac{k}{\ln\dfrac{z}{Z_{0h}}}\frac{k}{\ln\dfrac{z}{Z_{0m}}}(\bar{q}_v - q_{vsfc})\bar{u} \tag{5.39}$$

This is expressed by using the bulk coefficient of potential temperature defined by (5.32) as follows;

$$-\frac{k}{\ln\dfrac{z}{Z_{0h}}}\frac{k}{\ln\dfrac{z}{Z_{0m}}}(\bar{q}_v - q_{vsfc})\bar{u} = -C_h(\bar{q}_v - q_{vsfc})\bar{u} \tag{5.40}$$

Vertical flux of momentum, potential temperature and water vapor mixing ratio in the atmospheric boundary layer have been described with bulk coefficient. In the ground surface process of ***CReSS***, the exchange of sensible and latent heat between land and atmosphere is described by calculating these bulk coefficients. The effect of ground surface friction is included in a caluculation of momentum flux.

5.2 Calculations of Atmospheric Boundary Layer, Ground Surface Process and Soil Temperature

The ways of calculation used in JSM (Japan Spectral Model) by Segami et al. (1989) are basically adopted in the calculations of atmospheric boundary layer, ground surface process and soil temperature in ***CReSS***Their calculations are one-dimensional vertically. However, following main differences exist between the calculations in ***CReSS***and those in JSM.

- On the calculation of soil temperature, a four layer model is used in JSM but we can select an arbitrary number (m) of layers in ***CReSS***.
- On the calculation of soil temperature, it starts at $n-1$ step and uses a complicated calculation method adopting differential of flux on temperature in JSM. On the other hand in ***CReSS***, the calculation method is a simple implicit one, just solving simultaneous equations.
- Cloudiness is estimated from relative humidity by the method of Ohno and Isa (1983) in JSM, but it is estimated from mixing ratio of cloud water and cloud ice in ***CReSS***.
- In JSM, quantities of atmosphere are calculated at first and after that, sensible heat flux and latent heat flux on the ground are calculated from temperature and relative humidity of the atmosphere which are modified by the first calculation of the atmosphere. In ***CReSS***on the other hand, they are calculated from temperature and mixing ratio of water vapor of $n-1$ step and the result is applied for the atmosphere and the ground.
- The calculation of vertical diffusion is conducted for all layers of the atmosphere at every step in JSM and flux entering the atmosphere from the ground affects instantly (within the time step of Δt) to top of the atmosphere. In ***CReSS***, the layers affected by the flux can be designed arbitrary.
- On the calculation of vertical diffusion, an implicit method modifying dependent variables is used in JSM but an explicit method is used in ***CReSS***. Rate of change for time can be calculated and it is added as a forcing term for a time integration.

The calculations of the atmospheric boundary layer, ground surface process and soil temperature are conducted mainly as follows. They are independently conducted for the vertical one-dimension on each horizontal grid.

- Cloudiness is calculated. Usually cloudinesses are calculated for upper, middle and lower layers.
- Solar radiation and downward long wave radiation are calculated.

- Coefficients of fluxes of momentum, sensible heat and latent heat on the ground surface are calculated.
- Horizontal wind speed, potential temperature and mixing ratio of water vapor are modified by calculation of the atmospheric boundary layer process.
- Soil temperature is calculated.

It is notable that, on the ground surface process, flux is not calculated but its coefficient is calculated. The reason is that sensible and latent heat fluxes,

$$H_S = -\rho_a C_p C_h |V_a| (T_a - T_G) \tag{5.41}$$
$$_lE = -\rho_a \mathcal{L}_v C_h |V_a| \beta \left[q_{va} - q_{vs}(T_G)\right] \tag{5.42}$$

are given as external forces on the ground surface, and potential temperature flux [3] and flux of mixing ratio of water vapor,

$$F_\theta = -\rho_a C_h |V_a| (\theta_{va} - \theta_{vG}) \tag{5.43}$$
$$F_{q_v} = -\rho_a C_h |V_a| \beta \left[q_{va} - q_{vs}(T_G)\right] \tag{5.44}$$

are given as external forces from the ground surface in the atmosphere. Coefficients common to the two above $\rho_a C_h |V_a|$ (actually it is different between for potential temperature and for mixing ratio of water vapor) are calculated in the ground surface process and are given to the processes of the atmospheric boundary layer and soil temperature. Then sensible and latent heat fluxes are included in the process of soil temperature, while the effects of fluxes of potential temperature and mixing ratio of water vapor are included in the process of the atmospheric boundary layer. Mixing ratio of water vapor corresponding to the ground surface temperature is calculated in the ground surface process and given to each process simultaneously.

Processes for the calculation of the atmospheric boundary layer, the ground surface process and soil temperature are explained in the next section.

5.2.1 Cloudiness

The amount of solar radiation reaching to the ground decreases by the effect of cloud. Cloudiness is used to estimate this effect. The way of Ohno and Isa (1984) to estimate it by an empirical formula of cloudiness and relative humidity is explained in this section.

Cloudiness is nondimensional quantity from 0 to 1. Cloudiness is given as a function of relative humidity for upper, middle and lower layers, respectively. The height of each layer is not exact and 400hPa is for the upper layer, 500 and 700hPa for the middle layer and 850hPa for the lower layer in Ohno and Isa (1984). Upper layer corresponds to 7-7.5km, middle corresponds to 5-6 and 3km, and lower corresponds to 1.5km.

Cloudiness is given by 21 data every 5% ranging from 0 to 100% from the result of Ohno and Isa (1984). When average relative humidity in each layer is defined as $\overline{Rh}$[%], an integer k_n is defined as follows;

$$k_n = 1 + 20 \times \frac{\overline{Rh}}{100} \tag{5.45}$$

and cloudiness of lower layer CD_L、middle layer CD_M and upper layer CD_H is defined as follows (written in the format of `data` description of Fortran);

[3] Virtual potential temperature is used here instead of potential temperature. It makes no difference because flux is proportional to the difference of quantities. However, it is notable that it makes big difference when they are used to calculate CAPE.

```
      data cdldef / 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0,
     .              0.e0, 0.e0, 0.e0, 0.e0,.07e0,.11e0,.19e0,.40e0,
     .             .85e0, 1.e0, 1.e0, 1.e0, 1.e0 /

      data cdmdef / 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0,
     .              0.e0, 0.e0, 0.e0,.05e0,.12e0,.30e0,.40e0,.50e0,
     .             .70e0,.95e0, 1.e0, 1.e0, 1.e0 /

      data cdhdef / 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0, 0.e0,
     .              0.e0,.05e0,.15e0,.30e0,.55e0,.75e0,.87e0,.95e0,
     .              1.e0, 1.e0, 1.e0, 1.e0, 1.e0 /
```

To calculate relative humidity from mixing ratio of water vapor q_v and air temperature T_a, partial pressure of water vapor estimated from q_v and saturated water vapor pressure at the air temperature is calculated by using Tetens (1930)'s equation. Relative humidity is given from the ratio of the partial pressure of water vapor and saturated water vapor pressure.

Then, air pressure, air temperature and mixing ratio of water vapor are defined as $p\ (= \bar{p} + p')$, T_a and q_v, respectively, partial pressure of water vapor is given as follows;

$$e_a = \frac{q_v p}{\epsilon + (1-\epsilon)\, q_v} \tag{5.46}$$

where ϵ is the ratio of molecular mass of water vapor to that of dry air, which is 0.622 when mixing ratio of water vapor is provided with the unit of [kg kg^{-1}], and is 622.0 when it is provided with the unit of [g kg^{-1}]. The unit of air pressure p and partial pressure of water vapor e_a is [Pa].

We assume that air temperature T[°C] (note that the unit is degrees Celsius) is given, saturated water vapor pressure of the air temperature (Tetens (1930)'s equation) becomes

$$e_{as} = e_s \cdot 10^{\frac{a \cdot T}{b+T}} \tag{5.47}$$

or

$$e_{as} = e_s \exp\left(\frac{a \cdot T}{b+T} \ln 10\right) \tag{5.48}$$

where e_s (= 610.78 Pa) is saturated water vapor pressure at 0°C. The constant values a and b are defined as follows;

	a	b
$(T \geq 0°\mathrm{C})$ on the water	7.5	237.3
$(T < 0°\mathrm{C})$ on the ice	9.5	265.3

By using these, relative humidity Rh is gained as follows.

$$Rh = \frac{e_a}{e_{as}} \times 100 \tag{5.49}$$

Cloudiness is derived with this.

By using the method above, cloudiness is decided irrespective of the actual cloud distribution. Because mixing ratios of water substances are calculated in ***CReSS***, cloudiness should be estimated from these values. The total amount of cloud water [m^{-2}] is calculated as follows.

$$q_L = \int_{z_s}^{z_L} \rho\,(q_c + q_r + q_i + q_s + q_g)\,dz \tag{5.50}$$

$$q_M = \int_{z_L}^{z_M} \rho\,(q_c + q_r + q_i + q_s + q_g)\,dz \tag{5.51}$$

$$q_H = \int_{z_M}^{z_t} \rho\,(q_c + q_r + q_i + q_s + q_g)\,dz \tag{5.52}$$

where, $z_L,\ z_M$ are the heights of interfaces between upper, middle and lower layer, respectively. Then, cloudiness is estimated from the relation between these total amount of cloud water and the solar radiation[4].

5.2.2 Solar Radiation and Downward Long Wave Radiation

Solar radiation（short wave radiation） and downward long wave radiation from atmosphere are calculated as a function of cloudiness, zenith angle of the sun, near–surface temperature, water vapor pressure and albedo at the ground surface.

The estimation of solar radiation and atmospheric long wave radiation are summarized as follows.

- Partial pressure of water vapor e_a is obtained by the temperature and mixing ratio of water vapor q_v at the near surface layer (around the first or second layer).
- The zenith angle of the sun for each grid at the time is calculated.
- Downwarad short wave radiation is estimated by the partial pressure of water vapor e_a, zenith angle and cloud amount with the consideration for albedo.
- Downward long wave radiation is represented as a function of atmospheric temperature T_a, partial pressure of water vapor e_a and cloud amount.

Following subsections explain about net (downward) short wave radiation and downward long wave radiation, respectively.

Partial Pressure of Water Vapor at the Ground Surface

Partial pressure of water vapor at lower layer of atmosphere is estimated utilizing temperature and mixing ratio of water vapor at the lowest atmospheric layer or averaged among some lower atmospheric layers[5].

Zenith Angle of the Sun

Zenith angle of the sun is calculated as a function of the number of days from the first day of Janualy $jday$, local time T_{LC} and latitude at the point lat.

Local time is a function of longuitude based on the universal standard time UTC. It is expressed by

$$T_{LC} = UTC + lon/15 \tag{5.53}$$

[4]Kondo (2000)

[5]In JSM, averaged value of first and second atmospheric layers is used

Angle of the sun ϕ_s is expressed as

$$\phi_s = 23.44\cos(172 - jday) \tag{5.54}$$

then, zenith angle of the sun is expressed as follows (see fig.5.1):

$$\zeta = \sin(lat)\sin(\phi_s) + \cos(lat)\cos(\phi_s)\cos[0.2618(T_{LC} - 12)]. \tag{5.55}$$

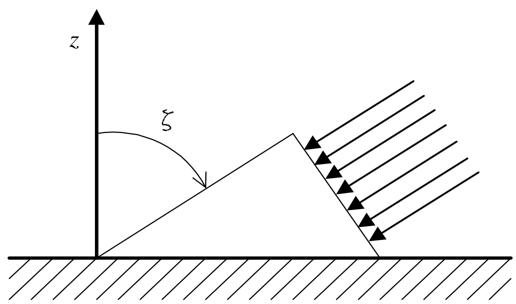

Figure 5.1. Zenith angle of the sunζ.

Net (Downward) Short Wave Radiation

The solar radiation which reaches at the ground surface is called downward short wave radiation. Considering it's reflection from the ground surface with a rate of albedo, downward short wave radiation which is absorbed into the ground is estimated. It is called net downward short wave solar radiation.

In order to estimate total solar radiation which reaches at the ground surface, it is needed to estimate solar radiation which arrive at the top of atmosphere S_∞[6].

$$S_\infty = S_0 \cos\zeta \qquad (S_0 = 1,367[\mathrm{W\ m^{-2}}] : \text{solar constant}) \tag{5.56}$$

where ζ is zenith angle of the sun. If we now let

$$a = 1.12 - b - 0.06\log_{10} e_a \qquad (1 \le e_a \le 3000[\mathrm{Pa}] : \text{water vapor pressure near surface}) \tag{5.57}$$

$$b = 0.43 + 0.00016 e_a \tag{5.58}$$

then solar radiation of the clear weather is

$$S^{\downarrow} = \begin{cases} S_\infty(a + b \times 10^{-0.13\sec\zeta}) & \cos\zeta > 0 \\ 0 & \cos\zeta \le 0 \end{cases} \tag{5.59}$$

Adding the effect of the cloud and its albedo, net downward short wave radiation which is absorbed into the ground is

$$RS^{\downarrow} = (1 - A_l)S^{\downarrow}(1 - cd_L \cdot \mathrm{CD}_L)(1 - cd_M \cdot \mathrm{CD}_M)(1 - cd_H \cdot \mathrm{CD}_H) \tag{5.60}$$

where

[6]in actual, $S_\infty = S_0(\bar{r}_E/r_E)^2\cos\zeta$. r_E is distance between earth and sun, $\bar{r}_E$ is its average

cd_L	absorbing and refrecting effects with the lower cloud	0.7
cd_M	absorbing and refrecting effects with the middle cloud	0.6
cd_H	absorbing and refrecting effects with the upper cloud	0.3

As mentioned before, a cloud amount is calculated by empirical formula using relative humidity (Ohno and Isa, 1984). Albedo A_l is shown in the following table.

land surface	given from data
sea ice or snow surface	constant to 0.6

At the sea surface, there is no need to consider solar radiation because sea temperature is constant during a calculation, so that there is no need to give the albedo.

Downward Long Wave Radiation from the Atmospere at the Ground Surface

Using empirical formula of Kondo (1976), downward long wave radiation from atmosphere including effects of cloud and water vapor is

$$L^{\downarrow} = \varepsilon_G \sigma T_a^4 [1 + (-0.49 + 0.0066\sqrt{e_a})(1 - \mathrm{CD} \cdot C \cdot A_c)] \tag{5.61}$$

where T_a is estimated by a temperature at the lowest atmospheric layer or averaged for some lower atmospheric layers[7] and

$$C = 0.75 - 0.005 e_a \tag{5.62}$$

$$A_c = \frac{\mathrm{CD}_L + 0.85\mathrm{CD}_M + 0.5\mathrm{CD}_H}{\mathrm{CD}} + 0.1\frac{N_r}{N} \tag{5.63}$$

$$\mathrm{CD} = \mathrm{CD}_L + \mathrm{CD}_M + \mathrm{CD}_H \tag{5.64}$$

[8] Then, utilized constants are expressed in the following table.

ε_G	emissivity for infrared region at the ground surface	0.95	
σ	Stefan-Boltzman constant	5.67×10^{-8}	W $m^{-2}K^{-4}$

5.2.3 Flux at the Ground Surface

Momentum flux at the ground surface τ_x, τ_y[kg m s^{-1} m^{-2} s^{-1} = N m^{-2}] is expressed as follows with bulk coefficient and the absolute value of wind speed.

$$\tau_x = \rho_a C_m |V_a| u_a \tag{5.65}$$

$$\tau_y = \rho_a C_m |V_a| v_a \tag{5.66}$$

Sendible and latent heat flux H_S, $_lE$ [J m^{-2} s^{-1} = W m^{-2}] is given as

[7] In JSM, averaged value of first and second atmospheric layers is used

[8] N_r/N means appearance rate of precipitating time. In temporal integration, it is set to 0 in case of no precipitation and to 1 in case of precipitation through the time. In JMS, on the other hand, it is set to 0 constantly though there is no confidence to be finest method.

$$H_S = -\rho_a C_p C_h |V_a| (T_a - T_G) \tag{5.67}$$

$${}_l E = -\rho_a \mathcal{L}_v C_h |V_a| \beta \left[q_{va} - q_{vs}(T_G) \right] \tag{5.68}$$

$$\tag{5.69}$$

where a and G mean the first layer of the atmosphere and the ground surface (first layer of the ground temperature), C_m and C_h are the bulk coefficient (with no dimension) of the momentum, and heat and water vapor, $q_{vs}(T_G)$ is saturated mixed ratio of ground temperature T_G, β is evapotranspiration coefficient and $\mathcal{L}_v$ is latent heat of evaporation of water. They are multiplied atmospheric density and rewrited to equivarent temperature flux F_θ [kg K m^{-2} s^{-1}] and water vapor mixed ratio flux F_{q_v}[kg m^{-2} s^{-1}] as

$$\begin{aligned} F_\theta = \frac{H_S}{C_p}\left(\frac{p_0}{p}\right)^{\frac{R}{C_p}} &= -\rho_a C_h |V_a| (\theta_{va} - \theta_{vG}) \\ &= -\rho_a u_* \theta_* \end{aligned} \tag{5.70}$$

$$\begin{aligned} F_{q_v} = \qquad E \qquad &= -\rho_a C_h |V_a| \beta \left[q_{va} - q_{vs}(T_G) \right] \\ &= -\rho_a u_* q_{v*} \end{aligned} \tag{5.71}$$

where θ_* is scale of potential temperature (frictional temperature) and q_{v*} is scale of mixing ratio (frictional mixing ratio). Friction velocity u_* is defined as

$$u_*^2 = C_m |V_a|^2 \tag{5.72}$$

where $|V_a|$ is absolute value of ground surface wind speed which expressed as

$$|V_a| = (u_a^2 + v_a^2)^{\frac{1}{2}} \tag{5.73}$$

Virtual potential temperature θ_v is

$$\theta_v = \frac{1 + \epsilon_{iv} q_v}{1 + q_v} \theta \tag{5.74}$$

where ϵ_{iv} is ratio of molecular weight of water vapor to that of dry air.

Non-dimensional bulk coefficients C_m, C_h are expressed by the scheme of Louis et al. (1980) considering momentum, heat and roughness parameter of water vapor as follows:

$$C_m = \left[\frac{k}{\ln\left(\frac{z_a}{Z_{0m}}\right)} \right]^2 f_m(Ri, z_a, Z_{0m}) \tag{5.75}$$

$$C_h = \frac{k}{\ln\left(\frac{z_a}{Z_{0m}}\right)} \frac{k}{\ln\left(\frac{z_a}{Z_{0h}}\right)} f_h(Ri, z_a, Z_{0m}, Z_{0h}) \tag{5.76}$$

where k (=0.4), z_a, Z_{0m}, Z_{0h} and Ri are Kalman coefficient, height of the first atmospheric layer, roughness of momentum, roughness of heat and water vapor, and Richardson number, respectively.

In unstable case ($Ri < 0$),

$$f_m = 1 - \frac{2b \cdot Ri}{1 + 3bc \left[\dfrac{k}{\ln\left(\dfrac{z_a}{Z_{0m}}\right)} \right]^2 \left(-\dfrac{Ri \cdot z_a}{Z_{0m}} \right)^{\frac{1}{2}}} \tag{5.77}$$

$$f_h = 1 - \frac{3b \cdot Ri}{1 + 3bc \dfrac{k}{\ln\left(\dfrac{z_a}{Z_{0m}}\right)} \dfrac{k}{\ln\left(\dfrac{z_a}{Z_{0h}}\right)} \left(-\dfrac{Ri \cdot z_a}{Z_{0h}} \right)^{\frac{1}{2}}} \tag{5.78}$$

$$b = c = 5 \tag{5.79}$$

In stable case ($Ri > 0$),

$$f_m = \frac{1}{1 + 2b \cdot Ri \cdot \sqrt{1 + d \cdot Ri}} \tag{5.80}$$

$$f_h = \frac{1}{1 + 3b \cdot Ri \cdot \sqrt{1 + d \cdot Ri}} \tag{5.81}$$

$$b = d = 5 \tag{5.82}$$

Roughness parameters of momentum Z_{0m} and of heat and water vapor Z_{0h} are shown in the following table. Roughness of the sea surface is refined at the every calculating step.

roughness	land surface	sea surface	sea ice or snow surface
Z_{0m}	given from data set	calculating as a function of u_*	optional constant value
Z_{0h}	constant value (0.1 m)	same with Z_{0m}	same with Z_{0m}

Standing on the above, actual calculation of land surface flux is shown as follows.

(1) Calculation of Absolute Value of Horizontal Wind Speed

Its absolute value $|V_a|$ is calculated by using horizontal wind speed at the first atmospheric layer (u_a, v_a) (**??**).

(2) Calculataion of Saturated Mixing Ratio of Water Vapor Corresponding to Ground Surface Temperature

Saturated mixing ratio of water vapor $q_{vs}(T_G)$ corresponding to ground temperature at the first ground layer or sea surface temperature is obtained by the equation of Tetens (1930) or (5.48).

From partial pressure of saturated water vapor by the equation of Tetens (1930), saturated mixing ratio of water vapor [kg/kg] is expressed as

$$q_{vs}(T_G) = \epsilon \frac{e_{as}}{p} \tag{5.83}$$

Originally this is the equation to define specific humidity, but we can regard it as mixing ratio with a small error (difference). p is expressed as $p = \bar{p} + p'$, and ϵ is ratio of molecular weight of dry air to water vapor.

(3) Definition of Mixing Ratio of Water Vapor at the Ground Surface

Mixing ratio of water vapor at the ground surface is defined for land, sea, sea ice and snow surfaces, respectively. Considering evapotranspirate efficiency WET which is given to land surface only, it is defined as follows:

for land surface,

$$q_{vsfc} = \beta \left[q_{vs}(T_G) - q_{va} \right] + q_{va} \tag{5.84}$$

for sea, sea ice and snow surfaces,

$$q_{vsfc} = q_{vs}(T_G) \tag{5.85}$$

(4) Calculation of Richardson Number

Richardson number of the ground surface is calculated in order to decide stability at the ground surface and to calculate f_m, f_h in the scheme of Louis et al. (1980).

Richardson number Ri can be expressed as

$$Ri = \frac{g z_a \Delta\theta_v}{\theta_{va}|V_a|^2} \tag{5.86}$$

where ϵ_{iv} indicate ratio of the molecular weight between dry air and water vapor, then

$$\theta_{vG} = T_G \frac{1 + \epsilon_{iv} q_{vsfc}}{1 + \ q_{vsfc}} \left(\frac{p_0}{p}\right)^{\frac{R}{C_p}} \tag{5.87}$$

$$\Delta\theta_v = \theta_{va} - \theta_{vG} \tag{5.88}$$

and z_a is the height of first atmospheric layer and g is the acceleration of gravity.

(5) Calculation of Z_{0m} and u_* at the Sea Surface at the First Step by Iteration

Though both roughnesses at the land surface Z_{0m} and Z_{0h} are constant value during the calculation, those at the sea surface are estimated by the iteration with frictional velocity considering its dependence on wind speed. The iteration method is used at the only first step and Z_{0m} is estimated by frictional velocity of the one step before; u_*^{n-1} on and after the second step.

Z_{0m} is expressed as a function of u_* based on the formula of Kondo (1975) as follows:

$$Z_{0m} = \ -34.7 \times 10^{-6} + 8.28 \times 10^{-4} u_* \qquad \text{for} \qquad u_* \leq 1.08[\text{m s}^{-1}] \tag{5.89}$$

$$Z_{0m} = -0.277 \times 10^{-2} + 3.39 \times 10^{-3} u_* \qquad \text{for} \qquad u_* > 1.08[\text{m s}^{-1}] \tag{5.90}$$

where maximum of Z_{0m} is 1.5×10^{-5} m. From (5.72), u_* would be

$$u_* = |V_a| \sqrt{C_m} \tag{5.91}$$

Although bulk coefficient C_m is estimated by (5.75), its coefficient f_m is depended on Richardson number which given by (5.86); in unstable case, it is estimated by (5.77) while in stable case, it by (5.80).

Z_{0m} is calculated with 5.89 or 5.90 using the friction velocity u_*. The calculation using the roughness is iterated untill the value of Z_{0m}, u_* converges. Then, we can decide Z_{0m}, u_* at the sea surface.

(6) Calculation of f_m, f_h for the Whole of a Region

In the subsection (5), the roughness parameter of momentum at the sea surface Z_{0m} is estimated. Roughness parameters of heat and water vapor Z_{0h} are defined as $Z_{0h} = Z_{0m}$. At the land surface, the roughness of momentum Z_{0m} is given as a data and the roughnesses of heat and water vapor Z_{0h} are defined as constant value 0.1 m. In case of sea ice and snow cover, roughnesses of momentum, heat and water vapor are defined as constant values.

Roughness parameters of momentum, heat and water vapor for the whole of a region are decided, so coefficients f_m, f_h which are used in the scheme of Louis et al. (1980) and multiply by bulk coefficient are calculated by using (5.77), (5.78), (5.80), and (5.81).

(7) Calculation of Bulk Coefficients C_m, C_h for the Whole of a Region

Using f_m, f_h and roughness parameters $Z_{0m} and Z_{0h}$ which are estimated from (5) and (6), bulk coefficients C_m, C_h are calculated with (5.75) and (5.76).

(8) Calculation of Friction Velocity u_* for the Whole of a Region

According to (5.91), friction velocity u_* is estimated for the whole of a region using bulk coefficients of momentum C_m and $|V_a|$.

(9) Calculation of Correction Term for Potential Temperature and Flux of Mixing Ratio Used in Kondo(1975).

The correction term by Kondo (1975) is estimated to calculate fluxes of water vapor and heat at the sea surface.

Kondo (1975) suggest the following equations of correction factor $B_h^{-1} and B_e^{-1}$ by the observation.

At the sea surface,

$$B_h^{-1} = \frac{1}{k} \ln \left(\frac{\nu + k u_* Z_{0m}}{\mathcal{D}_a} \right) \tag{5.92}$$

$$B_e^{-1} = \frac{1}{k} \ln \left(\frac{\nu + k u_* Z_{0m}}{\mathcal{D}_v} \right) = B_h^{-1} + \frac{1}{k} \ln \left(\frac{\mathcal{D}_a}{\mathcal{D}_v} \right) \tag{5.93}$$

or,

$$B_h^{-1} = \frac{1}{k} \ln \left(0.71 + 4.64 \times 10^4 k u_* Z_{0m} \right) \tag{5.94}$$

$$B_e^{-1} = B_h^{-1} - 0.168 \frac{1}{k} \tag{5.95}$$

At the land surface, sea ice and snow cover,

$$B_h^{-1} = 0 \tag{5.96}$$

$$B_e^{-1} = 0 \tag{5.97}$$

where ν is kinetic viscosity coefficient of air [m^2 s^{-1}], $\mathcal{D}_a$ is diffusion coefficient of air [m^2 s^{-1}] and $\mathcal{D}_v$ is diffusion coefficient of water vapor [m^2 s^{-1}][9]. [10]

[9] Kinetic viscosity coefficient of air ν[m^2 s^{-1}] is expressed by,

$$\nu = \nu_0 \frac{101325}{p} \left(\frac{T}{273.16} \right)^{1.754} \tag{5.98}$$

$$\nu_0 = 1.328 \times 10^{-5} \tag{5.99}$$

Diffusion coefficient of air $\mathcal{D}_a$[m^2 s^{-1}] is expressed by,

$$\mathcal{D}_a = \mathcal{D}_{a0} \frac{101325}{p} \left(\frac{T}{273.16} \right)^{1.78} \tag{5.100}$$

$$\mathcal{D}_{a0} = 1.87 \times 10^{-5} \tag{5.101}$$

Diffusion coefficient of water vapor $\mathcal{D}_v$[m^2 s^{-1}] is expressed by,

$$\mathcal{D}_v = \mathcal{D}_{v0} \frac{101325}{p} \left(\frac{T}{273.16} \right)^{1.81} \tag{5.102}$$

$$\mathcal{D}_{v0} = 2.23 \times 10^{-5} \tag{5.103}$$

However, it is enough to use apploximation in the text.

[10] B_h is Stanton number at the bottom layer and B_e is Dalton number at the bottom layer (大気科学講座 1 P95).

(10) Calculation of Flux Coefficients of Potential Temperature and Mixing Ratio of Water Vapor

In order to simplify the estimation of fluxes of potential temperature and mixing ratio of water vapor, the coefficients $E_\theta and E_{q_v}$ are calculated.

$$E_\theta = \frac{\theta_*}{\theta_{va} - \theta_{vG}} = \left[\frac{u_*}{C_h|V_a|} + B_h^{-1}\right]^{-1} \tag{5.104}$$

$$E_{q_v} = \frac{q_{v*}}{q_{va} - q_{vsfc}} = \left[\frac{u_*}{C_h|V_a|} + B_e^{-1}\right]^{-1} \tag{5.105}$$

Though same bulk coefficient C_h has been used in case of water vapor and heat fluxes, the use of above coefficients adopting $B_h^{-1} and B_e^{-1}$ make different coefficient value in case of the sea surface. In cases of the land surface, sea ice and snow cover, however, these values become equally 0.

(11) Calculation of Flux

Standing on the above, fluxes of momentum, sensible heat, latent heat, potential temperature and mixing ratio of water vapor at the ground surface are given as follows.

At first, absolute value of momentum flux is expressed by

$$\tau = \rho_a u_*^2 \tag{5.106}$$

and its x and y components are

$$\tau_x = \tau \frac{u_a}{|V_a|} \tag{5.107}$$

$$\tau_y = \tau \frac{v_a}{|V_a|} \tag{5.108}$$

Secondly, sensible heat and latent heat fluxes are expressed by

$$H_S = -\rho_a C_p u_* E_\theta (T_a - T_G) \tag{5.109}$$

$$_lE = -\rho_a \mathcal{L}_v u_* E_{q_v} (q_{va} - q_{vsfc}) \tag{5.110}$$

and, fluxes of potential temperature F_θ [kg K m^{-2} s^{-1}] and mixing ratio of water vapor F_{q_v} [kg m^{-2} s^{-1}] which are multiplied by atmospheric density are expressed as follows:

$$F_\theta = -\rho_a u_* E_\theta (\theta_{va} - \theta_{vG}) \tag{5.111}$$

$$F_{q_v} = -\rho_a u_* E_{q_v} (q_{va} - q_{vsfc}) \tag{5.112}$$

At land surface, $q_{va} - q_{vsfc} = \beta\,[q_{va} - q_{vs}(T_G)]$ is obtained using (5.84). Then, (5.110) and (5.112) take account of evapolation efficiency β through q_{vsfc}.

The above is the calculation of ground surface flux, however, (5.109) and (5.110) are used in the calculating process of ground temperature actually. (5.111) and (5.112) are used in the calculating process of atmosperic boundary layer. Therefore, this process returns following three coefficients which are common to these equations,

$$C_w = \rho_a u_*^2 \frac{1}{|V_a|} \tag{5.113}$$

$$C_\theta = \rho_a u_* E_\theta \tag{5.114}$$

$$C_{q_v} = \rho_a u_* E_{q_v} \tag{5.115}$$

and mixing ratio at the ground surface q_{vsfc} which include β in case of the land surface. Utilising these coefficients, x, y components of momentum flux and flux of mixing ratio are expressed as follows:

$$\tau_x = C_w u_a \tag{5.116}$$

$$\tau_y = C_w v_a \tag{5.117}$$

$$F_\theta = C_\theta(\theta_{va} - \theta_{vG}) \tag{5.118}$$

$$F_{q_v} = C_{q_v}(q_{va} - q_{vsfc}) \tag{5.119}$$

When we don't use an implicit method in the calculation of atmospheric boundary layer process, we can calculate fluxes of momentum, sensible heat, latent heat, potential temperatre and mixing ratio, instead.

(12) Calculation of Monitor Data at the Ground Surface

Temperature and wind speed at the ground surface are important to be compared with observed physical parameter at the ground surface. Surface boundary layer touching the ground surface is called as constant flux layer where vertical flux is constant regardless of height. According to the nature, temperature and mixing ratio of water vapor at a height of 1.5 m and wind speed at 10 m are calculated as a monitor value.

The method of calculation is simple. u_* is estimated previously and wind speed at a height of 10 m is calculated by (5.72) or (5.91) as

$$|V_{a10}| = \frac{u_*}{\sqrt{C_{m10}}} \tag{5.120}$$

C_{m10} is estimated by (5.75)–(5.81) in wnich $f_m and C_m$ are calculated with application of $z_a = 10$. Since wind direction is constant in vertical within the surface ground layer, x and y components are

$$u_{x10} = |V_{a10}| \frac{u_a}{|V_a|} \tag{5.121}$$

$$v_{y10} = |V_{a10}| \frac{v_a}{|V_a|} \tag{5.122}$$

In case of temperature and mixing ratio of water vapor, $z_a = 1.5$ is used to calculate $f_m and C_m$ by (5.75)–(5.81). Using the obtained bulk coefficient $C_{m1.5}$ and friction velocity u_* at a height of 1.5 m, wind speed at a height of 1.5 m is calculated as follows.

$$|V_{a1.5}| = \frac{u_*}{\sqrt{C_{m1.5}}} \tag{5.123}$$

Simultaneously, bulk coefficient $C_{h1.5}$ of potential temperature and mixing ratio at a height of 1.5m is calculated. Substituting these coefficients into (5.104) and (5.105), we get

$$E_{\theta 1.5} = \left[\frac{u_*}{C_{h1.5}|V_{a1.5}|} + B_h^{-1}\right]^{-1} \tag{5.124}$$

$$E_{q_v 1.5} = \left[\frac{u_*}{C_{h1.5}|V_{a1.5}|} + B_e^{-1}\right]^{-1} \tag{5.125}$$

Virtual potential temperature at a height of 1.5m $\theta_{v1.5}$ is related to that at the surface layer $\theta_{va} and at the ground \theta_{vG}$ as

$$\frac{\theta_{v1.5} - \theta_{vG}}{\theta_{va} - \theta_{vG}} = \frac{E_\theta}{E_{\theta 1.5}} \tag{5.126}$$

Then, vertual potential temperature at height of 1.5m is given by

$$\theta_{v1.5} = \theta_{vG} + (\theta_{va} - \theta_{vG})\frac{E_\theta}{E_{\theta 1.5}} \tag{5.127}$$

Similarly, mixing ratio of water vapor at height of 1.5m $q_{va1.5}$ is also related to that at the surface layer $q_{va} and at the ground q_{vsfc}$ as

$$\frac{q_{va1.5} - q_{vsfc}}{q_{va} - q_{vsfc}} = \frac{E_{q_v}}{E_{q_v 1.5}} \tag{5.128}$$

Then, mixing ratio of water vapor at a height of 1.5m $q_{va1.5}$ is given by

$$q_{va1.5} = q_{vsfc} + (q_{va} - q_{vsfc})\frac{E_{q_v}}{E_{q_v 1.5}} \tag{5.129}$$

where q_{vsfc} includes the effect of evapotranspiration coefficient.

(13) Calculation of the Roughness Parameter at the Sea Surface Z_{0m} for the Next Step

At the end of this calculation process, roughness Z_{0m} which relate to the momentum at the sea surface is calculated for the next step. Adopting the friction velocity u_* into (5.89) and (5.90), Z_{0m} at the sea surface is calculated and conserved untill the next step.

5.2.4 Atmospheric Boundery Layer Process

Momentum, sensible heat and latent heat which are given to the atmosphere from the ground surface are transported upward by the vertical diffusion. Although diffusion is caused by the turbulent flow, we should consider at some points for the diffusion in the boundary layer.

In CReSS, the vertical diffusion other than the usual turbulent flow is taken into consideration in the atmospheric boundary layer process. The height of boundary layer is specified by the user. Since vertical diffusion becomes so small that it is far from the ground surface, the height of which diffusion attains to should just be given. Vertical diffusion of the atmospheric boundary layer process is fundamentally calculated using level 2 of the turbulent closure model by Mellor and Yamada (1974) which is used in JSM (Segami et al. (1989)), and that is also calculation about only 1 dimension in vertical.

The equation of vertical diffusion is expressed with a z coordinate as follows.

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_m \frac{\partial u}{\partial z}\right) \tag{5.130}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_m \frac{\partial v}{\partial z}\right) \tag{5.131}$$

$$\frac{\partial \theta_v}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_h \frac{\partial \theta_v}{\partial z}\right) \tag{5.132}$$

$$\frac{\partial q_v}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_h \frac{\partial q_v}{\partial z}\right) \tag{5.133}$$

where the turbulent mixing coefficient added $K_0 = 1.0[\mathrm{m}^2 - \mathrm{s}^{-1}]$ was used. That is,

$$K_m = K_0 + l^2 \left| \frac{\partial \mathbf{V}}{\partial z} \right| S_m \tag{5.134}$$

$$K_h = K_0 + l^2 \left| \frac{\partial \mathbf{V}}{\partial z} \right| S_h \tag{5.135}$$

where S_m, S_h are the function of flux Richardson number (Mellor and Yamada, 1974) and length scale l is

$$l = \frac{kz}{1 + \dfrac{kz}{l_0}} \tag{5.136}$$

k is Karman's constant and l_0 is expressed by making E into turbulent energy as follows.

$$l_0 = 0.10 \times \frac{\displaystyle\int_{z_s}^{\infty} \rho E z dz}{\displaystyle\int_{z_s}^{\infty} \rho E dz} \tag{5.137}$$

Based on the above, the calculation method of boundary layer process is as follows.

(1) Calculation of Air Density between Grids

Air density ρ_k^w is calculated between each vertical grid which is needed in the case of difference specialization (see Fig. ref fig:pbl grid setting).

(2) Calculation of the square of vertical shear

Because of the use for following calculation of Richardson number, or of flux, the absolute value of vertical shear is calculated here.

$$\left| \frac{\partial \mathbf{V_k}}{\partial z} \right|^2 = \left(\frac{\partial u_k}{\partial z} \right)^2 + \left(\frac{\partial v_k}{\partial z} \right)^2 \tag{5.138}$$

(3) Gradient Richardson Number

In the level 2 by Mellor and Yamada (1974), flux Richardson number R_f is given as a function of gradient Richardson number R_i. It is defined as follows.

$$R_i = \frac{g\dfrac{\partial \theta_v}{\partial z}}{\bar{\theta}_v \left|\dfrac{\partial \mathbf{V_k}}{\partial z}\right|^2} \tag{5.139}$$

where $\bar{\theta}_v = \dfrac{1}{2}(\theta_{vk+1} + \theta_{vk})$.

(4) Flux Richardson Number

Following to Mellor and Yamada (1974), flux Richardson number R_f is given as follows.

$$R_f = 0.725\left(R_i + 0.186 - \sqrt{R_i^2 - 0.316R_i + 0.0346}\right) \tag{5.140}$$

(5) Calculation of $\tilde{S}_H and \tilde{S}_M$

The constants used by Mellor and Yamada (1974) are as follows.

A_1	A_2	B_1	B_2	C_1
0.78	0.78	15.0	8.0	0.056

Using these values, the following constants are estimated.

$$\gamma_1 \equiv \frac{1}{3} - \frac{2A_1}{B_1} = 0.2293333 \tag{5.141}$$

$$\gamma_2 \equiv \frac{B_2}{B_1} + \frac{6A_1}{B_1} = 0.8453333 \tag{5.142}$$

and,

$$\Gamma \equiv \frac{R_f}{1 - R_f} \tag{5.143}$$

Using these values, $\tilde{S}_H and \tilde{S}_M$ are expressed as follows.

$$\tilde{S}_H = 3A_2(\gamma_1 - \gamma_2\Gamma) \tag{5.144}$$

$$\tilde{S}_M = 3A_1(\gamma_1 - \gamma_2\Gamma)\frac{\gamma_1 - C_1 - (6A_1 + 3A_2)\dfrac{\Gamma}{B_1}}{\gamma_1 - \gamma_2\Gamma + 3A_1\dfrac{\Gamma}{B_1}} \tag{5.145}$$

These can be simplified more in the actual calculation. Eliminating Γ and considering $A_1 = A_2$, they can expresses as

$$\tilde{S}_H = 3A_2 \frac{\gamma_1 - (\gamma_1 + \gamma_2)R_f}{1 - R_f} \tag{5.146}$$

$$\tilde{S}_M = \tilde{S}_H \frac{X_1 - X_2 R_f}{X_3 - X_4 R_f} \tag{5.147}$$

The constants used here are as follows.

$$\gamma_1 + \gamma_2 = 1.074667 \tag{5.148}$$
$$3A_2 = 2.34 \tag{5.149}$$
$$X_1 = 0.173333 \tag{5.150}$$
$$X_2 = 0.641333 \tag{5.151}$$
$$X_3 = 0.229333 \tag{5.152}$$
$$X_4 = 0.918667 \tag{5.153}$$

(6)Calculation of $S_H and S_M$

Using $\tilde{S}_H and \tilde{S}_M$ which are calculated previously, $S_H and S_M$ are estimated. As the factor common to both $S_H and S_M$,

$$S_e = \sqrt{B_1(1 - R_f)\left|\frac{\partial \mathbf{V}}{\partial z}\right|^2 \tilde{S}_M} \tag{5.154}$$

is defined. Then, $S_H and S_M$ are expressed as follows.

$$S_H = S_e \tilde{S}_H \tag{5.155}$$
$$S_M = S_e \tilde{S}_M \tag{5.156}$$

(7)Calculation of Standard of a Length Scale l_0

In Mellor and Yamada (1974), standard of a length scale l_0 is calculated as a function of turbulent kinetic energy (5.137). A turbulent kinetic energy which is estimated in Chapter **??** "diffusion in sub-grid scale" is used.

(8)Calculation of a Length Scale l

Length scale l is given by (5.136) as follows.

$$l = \frac{z}{\frac{1}{k} + \frac{z}{l_0}} \tag{5.157}$$

(9)Calculation of $K_m and K_h$

Although $K_m and K_h$ are defined as (5.134) and (5.135), the absolute value of vertical shear is already applied to $S_M and S_H$ in the above-mentioned calculation. Then, they are rewritten as follows.

$$K_m = K_0 + l^2 S_M \tag{5.158}$$
$$K_h = K_0 + l^2 S_H \tag{5.159}$$

Here, $K_0 = 1.0$ [m^2 s^{-1}] .

(10)The Finite Difference Method for Equation of Vertical Diffusion

The vertical distributions of $K_m and K_h$ in boundary layer were calculated on the level between grids. Then, the vertical diffusion equations (5.130)–(5.133) are solved and a time changing rate and a correction value (integration value) are calculated in the cases of the explicit method and the inplicit method, respectively.

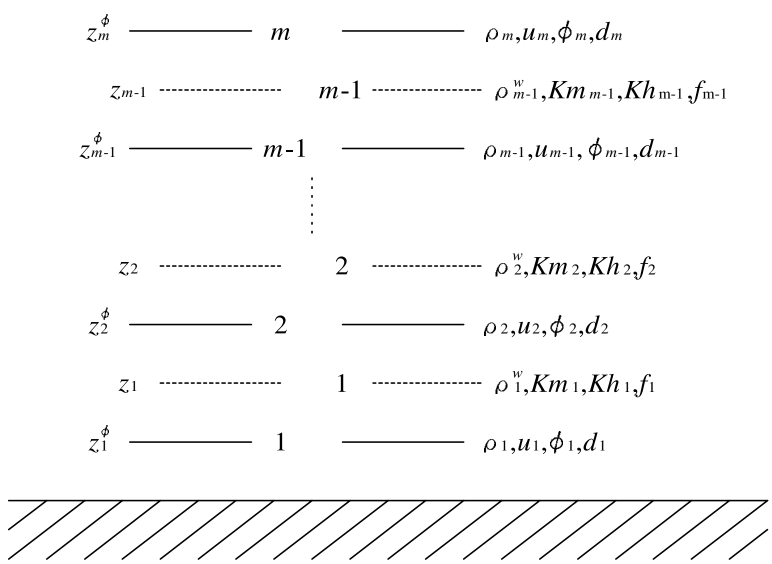

Figure 5.2. vertival grid setting used for atmospheric boundary layer process

We conduct a discretization for these equations with the arrangement as shown in Fig.5.2.

$$\frac{u_k^n - u_k^{n-1}}{\Delta t} = \frac{1}{\rho_k} \frac{1}{z_k - z_{k-1}} \left(\rho_k^w K_{m_k} \frac{u_{k+1}^N - u_k^N}{z_{k+1}^{\phi} - z_k^{\phi}} - \rho_{k-1}^w K_{m_{k-1}} \frac{u_k^N - u_{k-1}^N}{z_k^{\phi} - z_{k-1}^{\phi}} \right) \tag{5.160}$$

$$\frac{\phi_k^n - \phi_k^{n-1}}{\Delta t} = \frac{1}{\rho_k} \frac{1}{z_k - z_{k-1}} \left(\rho_k^w K_{m_k} \frac{\phi_{k+1}^N - \phi_k^N}{z_{k+1}^{\phi} - z_k^{\phi}} - \rho_{k-1}^w K_{m_{k-1}} \frac{\phi_k^N - \phi_{k-1}^N}{z_k^{\phi} - z_{k-1}^{\phi}} \right) \tag{5.161}$$

where u_k^n and ϕ_k^n (subscript with bottom k represents a vertical grid number and that with top n represents the time step) are used as a representative of the velocity component and of the scalar, respectively. N which appears in the right-hand side of difference equation becomes discretization of the explicit method in the case of $N = n - 1$, and becomes discretization of the implicit method in the case of $N = n$.

Coefficients of surface fluxes; C_w in (5.113), C_θ in (5.114) and C_{q_v} in (5.115) are used at the bottom layer ($k = 1$).

$$\frac{u_1^n - u_1^{n-1}}{\Delta t} = \frac{1}{\rho_1} \frac{1}{z_1} \left[\rho_1^w K_{m_1} \frac{u_2^N - u_1^N}{z_2^{\phi} - z_1^{\phi}} - C_w (u_1^N - u_{sfc}^N) \right] \tag{5.162}$$

$$\frac{\phi_1^n - \phi_1^{n-1}}{\Delta t} = \frac{1}{\rho_1} \frac{1}{z_1} \left[\rho_1^w K_{m_1} \frac{\phi_2^N - \phi_1^N}{z_2^{\phi} - z_1^{\phi}} - C_\phi (\phi_1^N - \phi_{sfc}^N) \right] \tag{5.163}$$

where

$$u_{sfc}^N = 0 \tag{5.164}$$

and representative of the scalar is ϕ. In these equations, $z_0 = 0$ can define the forrowing signs,

$$d_k = \frac{1}{\rho_k} \frac{1}{z_k - z_{k-1}} \tag{5.165}$$

$$f_k = \rho_k^w K_{m_k} \frac{1}{z_{k+1}^\phi - z_k^\phi} \tag{5.166}$$

We can understand K_{h_k} using f_k .
In the case of $k = 1$,

$$\frac{u_1^n - u_1^{n-1}}{\Delta t} = d_1 \left[f_1(u_2^N - u_1^N) - C_w u_1^N \right] \tag{5.167}$$

$$\frac{\phi_1^n - \phi_1^{n-1}}{\Delta t} = d_1 \left[f_1(\phi_2^N - \phi_1^N) - C_\phi(\phi_1^N - \phi_{sfc}^N) \right] \tag{5.168}$$

In the case of $k \geq 2$,

$$\frac{u_k^n - u_k^{n-1}}{\Delta t} = d_k \left[f_k(u_{k+1}^N - u_k^N) - f_{k-1}(u_k^N - u_{k-1}^N) \right] \tag{5.169}$$

$$\frac{\phi_k^n - \phi_k^{n-1}}{\Delta t} = d_k \left[f_k(\phi_{k+1}^N - \phi_k^N) - f_{k-1}(\phi_k^N - \phi_{k-1}^N) \right] \tag{5.170}$$

About $k = m - 1$,

$$\frac{u_{m-1}^n - u_{m-1}^{n-1}}{\Delta t} = d_{m-1} \left[f_{m-1}(u_m^{n-1} - u_{m-1}^N) - f_{m-2}(u_{m-1}^N - u_{m-2}^N) \right] \tag{5.171}$$

$$\frac{\phi_{m-1}^n - \phi_{m-1}^{n-1}}{\Delta t} = d_{m-1} \left[f_{m-1}(\phi_m^{n-1} - \phi_{m-1}^N) - f_{m-2}(\phi_{m-1}^N - \phi_{m-2}^N) \right] \tag{5.172}$$

where since u_m^{n-1} and ϕ_m^{n-1} in the case of $k = m - 1$ are given as boundary condition, this process is eternal.

In the case of $N = n - 1$, these equations are solved by the explicit method and time change rate is acquired. In the case of $N = n$, they are solved by the implicit method and integration value is acquired.

5.2.5 Ground Temperature

The ground temperature is calculated by the model with m layers.

Figure 5.3 shows a ground grid setting used for calculation of ground temperature. In the calculation, only the vertical heat conduction is taken into consideration, while the horizontal heat diffusion does not take into consideration.

Here, time evolution equation system of the ground temperature in the model with m layer is expressed as follows.

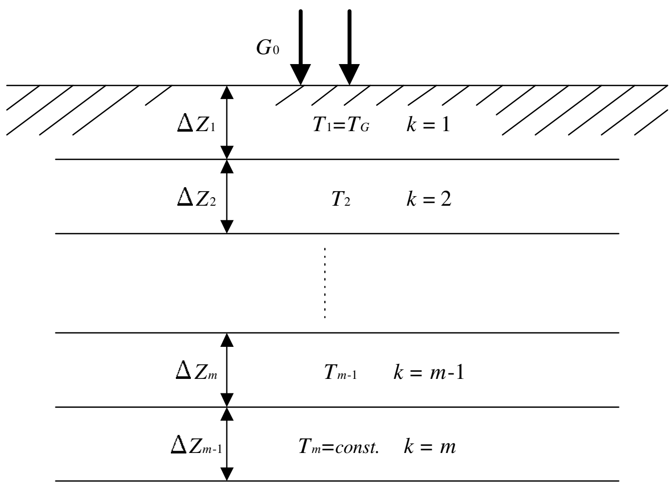

Figure 5.3. A underground setup used for calculation of ground temperature.

$$\begin{aligned}
\frac{\partial T_1}{\partial t} &= \frac{G_0}{\rho_G C_G \Delta Z_1} && + \frac{2\nu_G}{\Delta Z_1(\Delta Z_2+\Delta Z_1)}(T_2-T_1) \\
\frac{\partial T_2}{\partial t} &= -\frac{2\nu_G}{\Delta Z_2(\Delta Z_2+\Delta Z_1)}(T_2-T_1) && + \frac{2\nu_G}{\Delta Z_2(\Delta Z_3+\Delta Z_2)}(T_3-T_2) \\
&\cdots\cdots\cdot \\
\frac{\partial T_k}{\partial t} &= -\frac{2\nu_G(T_k-T_{k-1})}{\Delta Z_k(\Delta Z_k+\Delta Z_{k-1})} && + \frac{2\nu_G(T_{k+1}-T_k)}{\Delta Z_k(\Delta Z_{k+1}+\Delta Z_k)} \\
&\cdots\cdots\cdot \\
\frac{\partial T_{m-1}}{\partial t} &= -\frac{2\nu_G(T_{m-1}-T_{m-2})}{\Delta Z_{m-1}(\Delta Z_{m-1}+\Delta Z_{m-2})} && + \frac{2\nu_G(T_m-T_{m-1})}{\Delta Z_{m-1}(\Delta Z_m+\Delta Z_{m-1})}
\end{aligned} \tag{5.173}$$

In the m layer, temperature T_m is constant during a calculation period. Heat capacitiy of the ground per unit volume $\rho_G C_G$ and heat diffusion coefficient ν_G are

$$\rho_G C_G = 2.3\times 10^6 \qquad [\mathrm{Jm^{-3}K^{-1}}] \tag{5.174}$$
$$\nu_G = 7.0\times 10^{-7} \qquad [\mathrm{m^2s^{-1}}] \tag{5.175}$$

In (5.173) , G_0 which is appeared in the first ground layer ($k = 1$) is heat flux which goes to the ground. It is sum of net radiation R_{net}, sensible heat H_S and latent heat ${}_lE$.

$$G_0 = R_{net} - H_S - {}_lE \tag{5.176}$$

They are given as follows, respectively.

$$R_{net} = RS^{\downarrow} + L^{\downarrow} - L^{\uparrow} \tag{5.177}$$
$$H_S = -C_p C_\theta (T_a - T_G) \tag{5.178}$$
$${}_lE = -\mathcal{L}_v C_{q_v} \beta \left[q_{va} - q_{vs}(T_G)\right] \tag{5.179}$$

where the last term of net radiation R_{net} shows net upward longwave radiation $T_1 = T_G$ and,

$$L^{\uparrow} = \varepsilon_G \sigma T_G^4 \tag{5.180}$$

β in the equation of latent heat ${}_lE$ is evapotranspiration coefficient which is supposed to take constant value during the integration period. Then, radiation process is used only for calculation of the heat balance at the ground surface.

Time integration of the ground temperature solved by implicit scheme. In order to disperse the time evolution equation of ground temperature (5.173),

about $k = 1 \sim m-1$, the followings are defined.

$$a_1 = 0 \tag{5.181}$$

$$a_k = -\Delta t \frac{2\nu_G}{\Delta z_k(\Delta z_k + \Delta z_{k-1})} \tag{5.182}$$

$$b_k = -\Delta t \frac{2\nu_G}{\Delta z_k(\Delta z_{k+1} + \Delta z_k)} \tag{5.183}$$

Then, (5.173) is dispersed as follows.

$$\begin{aligned}
T_1^n &= T_1^{n-1} + \frac{\Delta t G_0}{\rho_G C_G \Delta Z_1} + b_1(T_1^n - T_2^n) \\
T_2^n &= T_2^{n-1} + a_2(T_2^n - T_1^n) + b_2(T_2^n - T_3^n) \\
&\cdots\cdots\cdot \\
T_k^n &= T_k^{n-1} + a_k(T_k^n - T_{k-1}^n) + b_k(T_k^n - T_{k+1}^n) \\
&\cdots\cdots\cdot \\
T_{m-1}^n &= T_{m-1}^{n-1} + a_{m-1}(T_{m-1}^n - T_{m-2}^n) + b_{m-1}(T_{m-1}^n - T_m)
\end{aligned} \tag{5.184}$$

The simultaneous equations can be expressed using a procession as follows.

$$\begin{aligned}
\begin{pmatrix} T_1^n \\ T_2^n \\ \vdots \\ T_k^n \\ \vdots \\ T_{m-1}^n \end{pmatrix} &= \begin{pmatrix} T_1^{n-1} + \dfrac{\Delta t G_0}{\rho_G C_G \Delta Z_1} \\ T_2^{n-1} \\ \vdots \\ T_k^{n-1} \\ \vdots \\ T_{m-1}^{n-1} - b_{m-1} T_m \end{pmatrix} \\
&+ \begin{pmatrix} b_1 & -b_1 & 0 & \cdots & \cdots & \cdots & 0 \\ -a_2 & a_2 + b_2 & -b_2 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -a_k & a_k + b_k & -b_k & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & -a_{m-1} & a_{m-1} + b_{m-1} \end{pmatrix} \begin{pmatrix} T_1^n \\ T_2^n \\ \vdots \\ T_k^n \\ \vdots \\ T_{m-1}^n \end{pmatrix}
\end{aligned} \tag{5.185}$$

Furthermore, it can be expressed briefly like

$$\mathbf{T}^n = \mathbf{F} + A\mathbf{T}^n \tag{5.186}$$

If the term of $\mathbf{T}^n$ is shifted using unit procession I of $(m-1) \times (m-1)$,

$$(I - A)\mathbf{T}^n = \mathbf{F} \tag{5.187}$$

is obtained. While it can be solved easily by the elimination of a gauss (refer to the 6.1.3 for explanation of a concrete solution method), ground temperature of all layers at the time of n are calculated.